PULSE TRANSFER FUNCTION AND MANIPULATION OF BLOCK DIAGRAMS – Closed loop systems

Digital control

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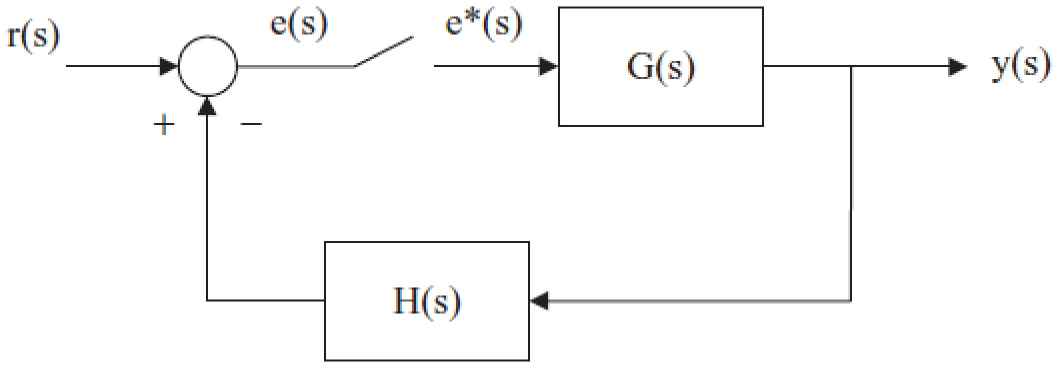
Closed-loop systems



The block diagram of a closed-loop sampled data system is shown in Figure 6.28. Derive an expression for the transfer function of the system.

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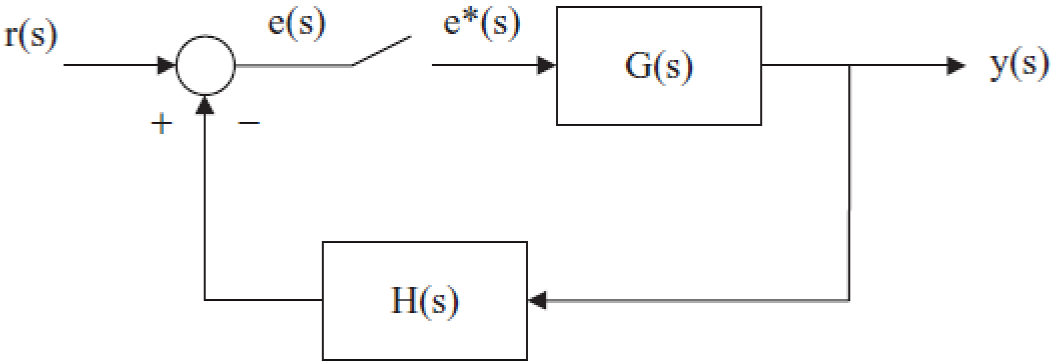

Solution

For the system in Figure 6.28 we can write

$$e(s) = r(s) - H(s)y(s) \tag{6.38}$$

and

$$y(s) = e^*(s)G(s). \tag{6.39}$$

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Substituting (6.39) into (6.38),

$$e(s) = r(s) - G(s)H(s)e^*(s) \tag{6.40}$$

or

$$e^*(s) = r^*(s) - GH^*(s)e^*(s)$$

and, solving for $e^*(s)$, we obtain

$$e^*(s) = \frac{r^*(s)}{1 + GH^*(s)} \tag{6.41}$$

and, from (6.39),

$$y(s) = G(s)\frac{r^*(s)}{1 + GH^*(s)}. \tag{6.42}$$

The sampled output is then

$$y^*(s) = \frac{r^*(s)G^*(s)}{1 + GH^*(s)} \tag{6.43}$$

Writing (6.43) in z-transform format,

$$y(z) = \frac{r(z)G(z)}{1 + GH(z)} \tag{6.44}$$

and the transfer function is given by

$$\frac{y(z)}{r(z)} = \frac{G(z)}{1 + GH(z)}. \tag{6.45}$$

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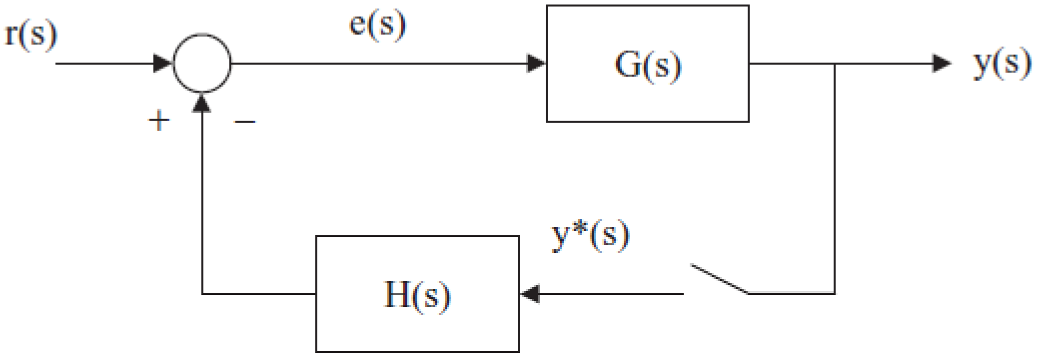

Figure 6.29 Closed-loop sampled data system

The block diagram of a closed-loop sampled data system is shown in Figure 6.29. Derive an expression for the output function of the system.

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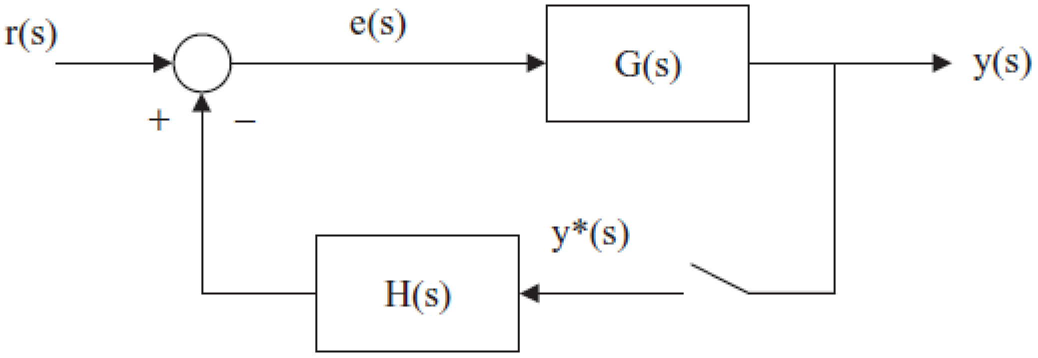

Figure 6.29 Closed-loop sampled data system

Solution

For the system in Figure 6.29 we can write

$$y(s) = e(s)G(s) \tag{6.46}$$

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and

$$e(s) = r(s) - H(s)y^*(s). \tag{6.47}$$

Substituting (6.47) into (6.46), we obtain

$$y(s) = G(s)r(s) - G(s)H(s)y^*(s) \tag{6.48}$$

or

$$y^*(s) = Gr^*(s) - GH^*(s)y^*(s). \tag{6.49}$$

Solving for $y^*(s)$, we obtain

$$y^*(s) = \frac{Gr^*(s)}{1 + GH^*(s)} \tag{6.50}$$

and

$$y(z) = \frac{Gr(z)}{1 + GH(z)}. \tag{6.51}$$

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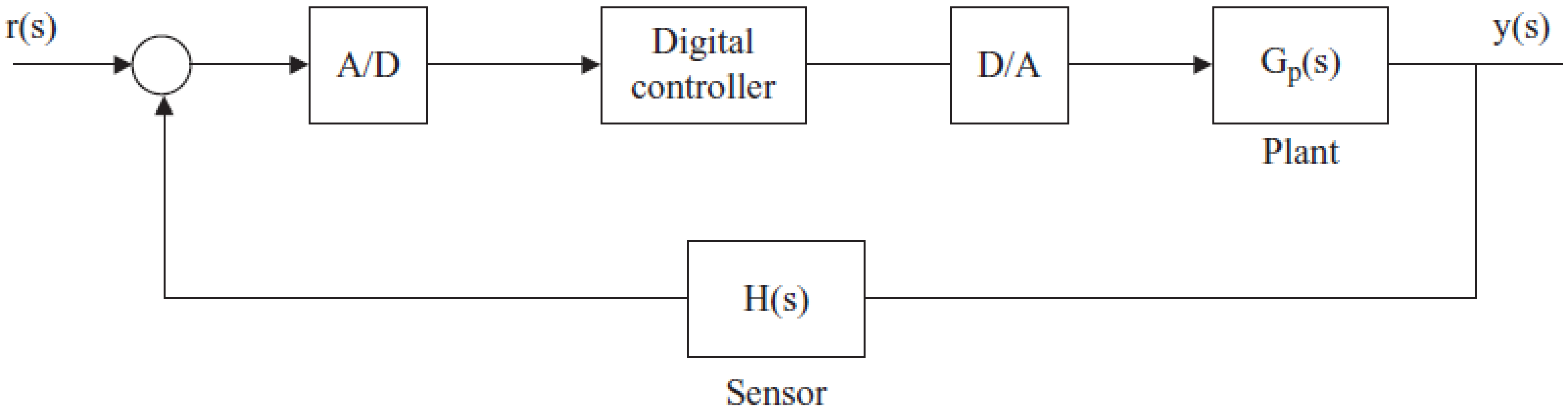

Figure 6.30 Closed-loop sampled data system

The block diagram of a closed-loop sampled data control system is shown in Figure 6.30. Derive an expression for the transfer function of the system.

Closed-loop systems

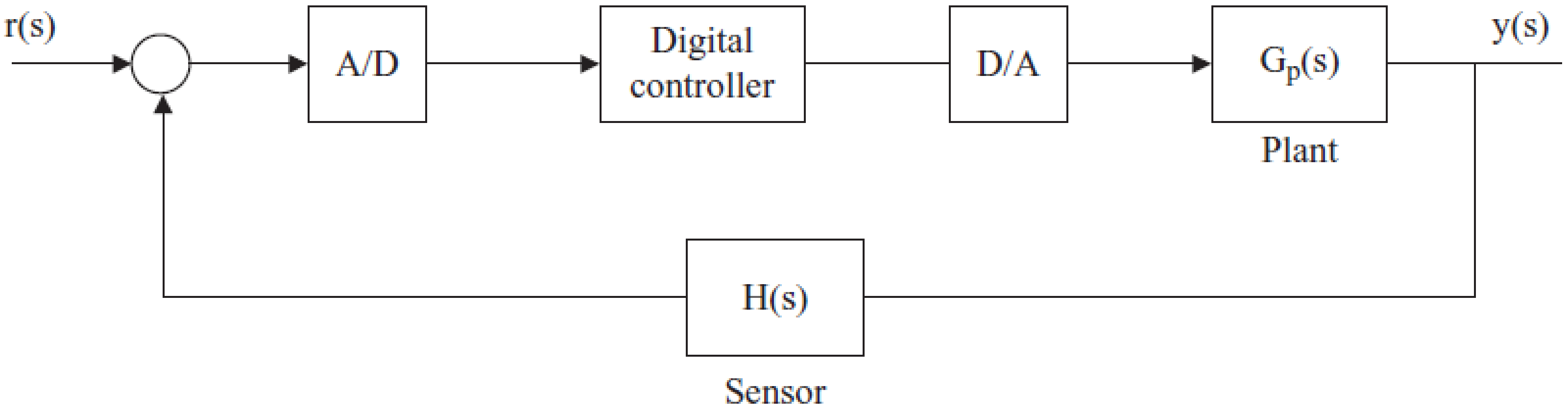

Figure 6.30 Closed-loop sampled data system

Solution

The A/D converter can be approximated with an ideal sampler. Similarly, the D/A converter at the output of the digital controller can be approximated with a zero-order hold. Denoting the digital controller by $D(s)$ and combining the zero-order hold and the plant into $G(s)$, the block diagram of the system can be drawn as in Figure 6.31. For this system can write

$$e(s) = r(s) - H(s)y(s) \tag{6.52}$$

and

$$y(s) = e^*(s)D^*(s)G(s). \tag{6.53}$$

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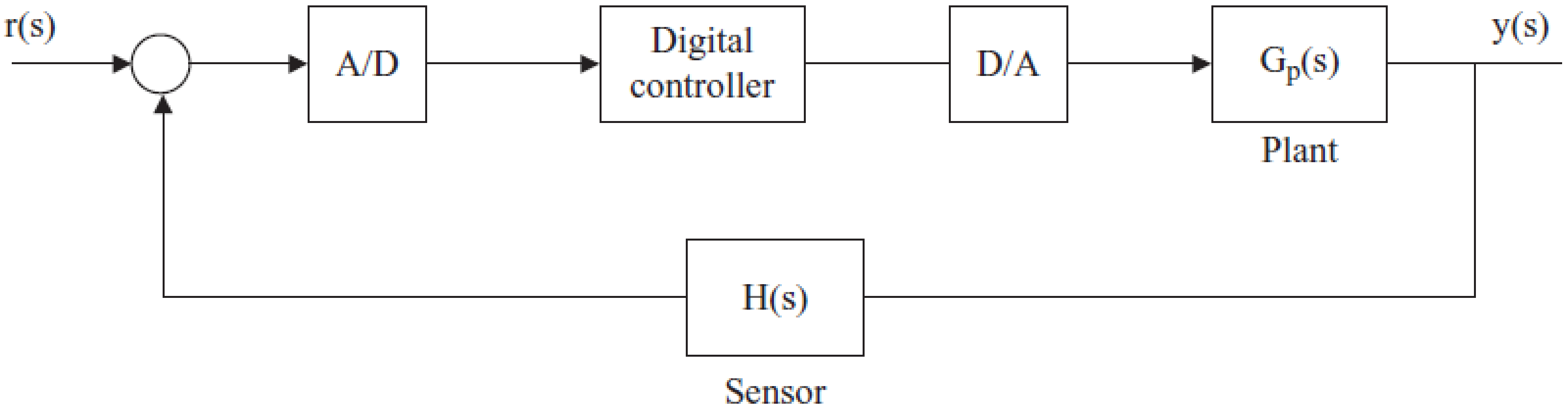

Figure 6.30 Closed-loop sampled data system

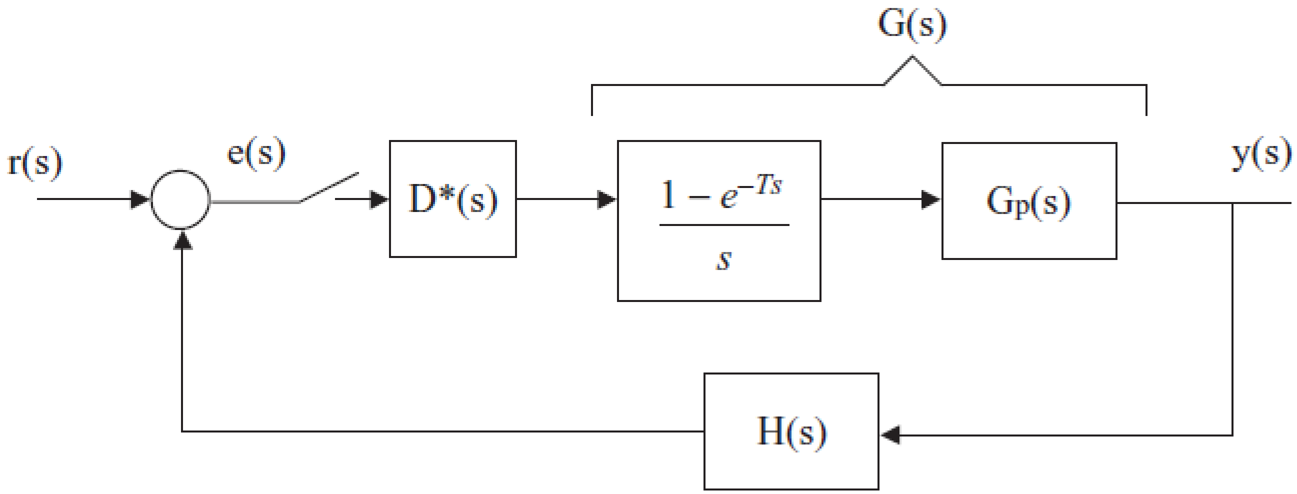

Figure 6.31 Equivalent diagram for Example 6.23

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Note that the digital computer is represented as $D^*(s)$. Using the above two equations, we can write

$$e(s) = r(s) - D^*(s)G(s)H(s)e^*(s) \tag{6.54}$$

or

$$e^*(s) = r^*(s) - D^*(s)GH^*(s)e^*(s)$$

and, solving for $e^*(s)$, we obtain

$$e^*(s) = \frac{r^*(s)}{1 + D^*(s)GH^*(s)} \tag{6.55}$$

and, from (6.53),

$$y(s) = D^*(s)G(s)\frac{r^*(s)}{1 + D^*(s)GH^*(s)}. \tag{6.56}$$

The sampled output is then

$$y^*(s) = \frac{r^*(s)D^*(s)G^*(s)}{1 + D^*(s)GH^*(s)}, \tag{6.57}$$

Writing (6.57) in z-transform format,

$$y(z) = \frac{r(z)D(z)G(z)}{1 + D(z)GH(z)} \tag{6.58}$$

and the transfer function is given by

$$\frac{y(z)}{r(z)} = \frac{D(z)G(z)}{1 + D(z)GH(z)}. \tag{6.59}$$

Closed-Loop Time Response

The closed-loop time response of a sampled data system can be obtained by finding the inverse z-transform of the output function.

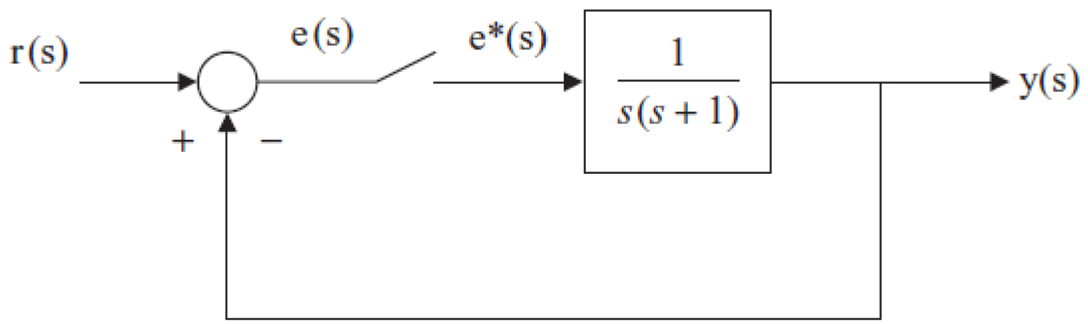

Figure 6.32 Closed-loop system

Example 6.24

A unit step signal is applied to the sampled data digital system shown in Figure 6.32. Calculate and plot the output response of the system. Assume that $T = 1$ s.

Closed-Loop Time Response

Solution

The output response of this system is given in (6.44) as

$$y(z) = \frac{r(z)G(z)}{1 + GH(z)}.$$

where

$$r(z) = \frac{z}{z-1}, \quad G(z) = \frac{z(1-e^{-T})}{(z-1)(z-e^{-T})}, \quad H(z) = 1;$$

thus,

$$y(z) = \frac{z/z-1}{1 + (z(1-e^{-T})/(z-1)(z-e^{-T}))} \frac{z(1-e^{-T})}{(z-1)(z-e^{-T})}.$$

Simplifying,

$$y(z) = \frac{z^2(1-e^{-T})}{(z^2 - 2ze^{-T} + e^{-T})(z-1)}.$$

Closed-Loop Time Response

Since $T = 1$,

$$y(z) = \frac{0.632z^2}{z^3 - 1.736z^2 + 1.104z - 0.368}.$$

After long division we obtain the first few terms

$$y(z) = 0.632z^{-1} + 1.096z^{-2} + 1.25z^{-3} + \ldots.$$

The first 10 samples of the output response are shown in Figure 6.33.